Spring 2014 **Math 601** Name: ______________________

Quiz 2

Question 1. (10 pts)

Solve the following linear system

$$\begin{cases} 2x + 8y + 4z = 2 \\ 2x + 5y + z = 5 \\ 4x + 10y - z = 1 \end{cases}$$

Solution: Set up the augmented coefficient matrix

$$\left[\begin{array}{ccc|c} 2 & 8 & 4 & 2 \\ 2 & 5 & 1 & 5 \\ 4 & 10 & -1 & 1 \end{array}\right]$$

change it to its echelon form

$$\left[\begin{array}{ccc|c} 1 & 4 & 2 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3 & 9 \end{array}\right]$$

So

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -4 \\ 3 \end{bmatrix}$$

is a solution.

Question 2. (5 pts)

Recall that a complex matrix is called unitary if $AA^* = I$, where $A^* = (\overline{A})^T$. Check whether $B = \begin{bmatrix} \frac{1+i}{2} & \frac{1-i}{2} \\ \frac{1-i}{2} & \frac{1+i}{2} \end{bmatrix}$ is unitary.

Solution: A direct calculation shows that $BB^* = I$, hence B is unitary.

Question 3. (5 pts)

Use the fact $\operatorname{tr}(AB) = \operatorname{tr}(BA)$ to show that

$$\operatorname{tr}(CAC^{-1}) = \operatorname{tr}(A).$$

Here all matrices are square matrices and C is invertible.

Solution:

$$\operatorname{tr}(CAC^{-1}) = \operatorname{tr}(C^{-1}(CA)) = \operatorname{tr}((C^{-1}C)A) = \operatorname{tr}(IA) = \operatorname{tr}(A)$$